Convex Storage Loss Modeling for Optimal Energy Management

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Outline of the presentation

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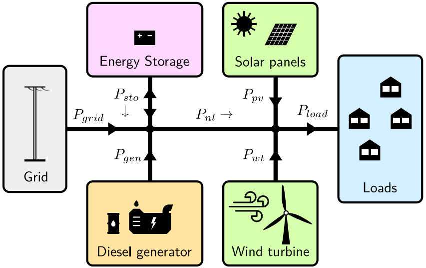
Energy management (EM) is often optimization based

EM = control of the power flows in a system with *energy storages*



Control objectives of EM:

- Minimizing a criterion (economical, ecological…)
- Satisfaying constraints (e.g. storage bounds)

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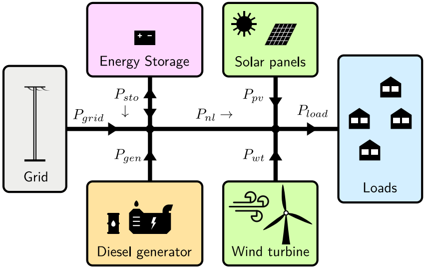

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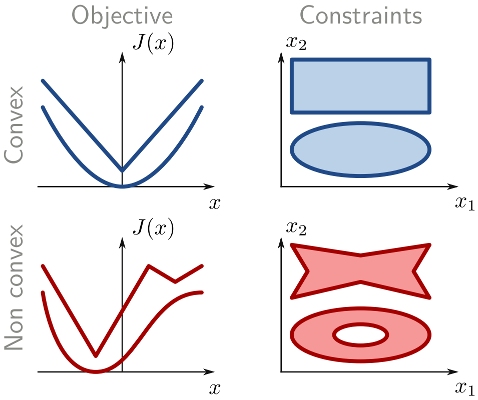
EM is often based on *online* optimization (ex.: Model Predictive Control)
→ convergence should be *reliable* → optim. problem should be **convex**

Convex optimization problems can be solved reliably



Optimization problem:

$$\min_{x \in \mathbb{R}^n} J(x), \text{ s.t. } g(x) \leq 0, \; h(x) = 0$$

Convexity conditions:

- Objective function J: convex
- Inequality function g: convex
- Equality function h: **linear**

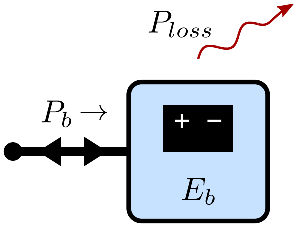
Outline of the presentation

Generic storage model

Storage dynamics with losses is *linear:*

$$E_b(k+1) = E_b(k) + (P_b(k) - P_{loss})\Delta_t$$

$\longrightarrow$ Convexity of the storage model depends on the *convexity of the loss expression*: “$P_{loss} = ...$”



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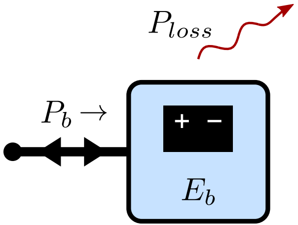
Trade-off in the choice of the loss expression:

- Convex for efficient optimization
- Physically realistic



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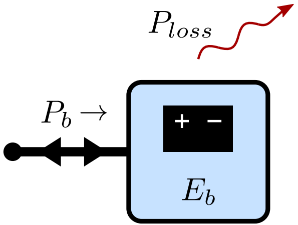

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Unfortunate limitation

- Only *linear* loss expressions are genuinely convex, but *very limiting* (see article)
- Many reasonable expressions are *not convex* (Joule heating: $P_{loss} \propto P_b^2$)

Relaxation of storage losses

Key idea (widely used in literature)

Relax the equality constraint (losses = some expression) to an inequality (losses ≥ some expression).

⟶ This allows using *any convex expression* for losses

Generic loss formulation (assuming a dependency on storage power and energy):

$$P_{loss} \geq g(P_b, E_b)$$

where g means any *convex* function.

Applicability of the relaxation of losses

In many applications, the inequality will be *tight at the optimum*:

$$\text{“}P_{loss} \geq g(P_b, E_b)\text{”} \quad \longrightarrow \quad \text{“}P^*_{loss} = g(P^*_b, E^*_b)\text{”}$$

Heuristic justification: “Positive price argument”

If the incremental cost of wasting energy is *positive*,
then, at the optimum, no energy should be wasted.

(see article for the few references giving detailed mathematical conditions for exact relaxation)

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Case where loss relaxation cannot work: negative energy price

Any system where the storage should *absorb an excess of energy* which cannot be dissipated for free, like in some grid congestions.

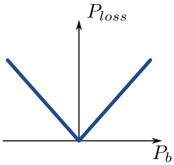
Consequence: there can be an *artificial excess* of storage loss (excess: $P_{loss} - g > 0$).

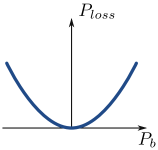
Outline of the presentation

Possibilities for loss expressions

Existing convex loss expressions (beyond linear):

- Piecewise-linear-in-P : $g(P_b, E_b) = c_+ P_b^+ + c_- P_b^-$
  Physics-free, widespread usage
  ($\longrightarrow$ relates to **constant efficiency** storage model (see article))
- Quadratic-in-P: $g(P_b, E_b) = \rho . P_b^2$
  approx. Joule heating ($r.I^2$) when Open Circuit Voltage is constant
- Quadratic-in-P over linear-in-E (P^2/E):
  approx. Joule heating in a capacitor (OVC $\propto E$)





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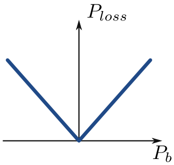
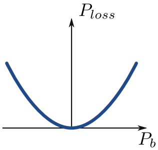




Our proposition: the "convex monomial loss model" ($\sim P^a/E^b$)

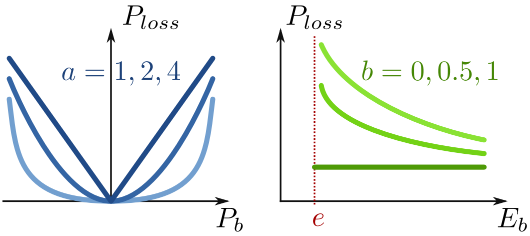
One continous family of nonlinear convex loss model:

- contains all existing models as special cases
- parametrized by 4 (or 8) real coefficients:
  suitable for *model fitting* to experimental loss data

Contribution: the “Convex monomial loss model”

Loss expression (symmetric charge/discharge case):

$$g(P_b, E_b) = c\frac{|P_b|^a}{|E_b - e|^b}$$



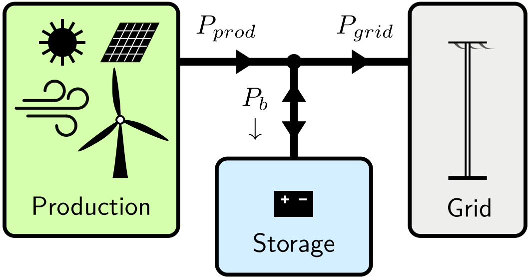
Convex with $a \geq 1$, $b \geq 0$ and $b \leq a - 1$ (see article)

Examples:

- Quad-in-P ($a = 2$) $\longrightarrow$ $b \in [0, 1]$, e.g. P^2/E (capacitor)
- PWL-in-P ($a = 1$) $\longrightarrow$ $b = 0$ ($\longrightarrow$ no SoE dependency allowed!)

Outline of the presentation

Application: storage for grid-connected production



Optimization objective: maximize value of energy sold to grid at price c_{grid}:

$$\max C_{grid} = \sum_{k=1}^{K} c_{grid}(k).P_{grid}(k)\Delta_t$$

Code available in Jupyter notebook `https://github.com/pierre-haessig/convex-storage-loss`

Scenario description: production shifting

Parameters: 2 hours ($K = 20, \Delta_t = 0.1\,\mathrm{h}$), $E_{rated} = 1\,\mathrm{kWh}$ storage

1. 1st hour: prod. $P_{prod} = 1\,\mathrm{kW}$ and *low* price $c_{grid} = 0.1\,€/\mathrm{kWh}$
2. 2nd hour: *zero* production and *high* price $c_{grid} = 0.2\,€/\mathrm{kWh}$

Interpretation

Storage can shift the production to the high price hour

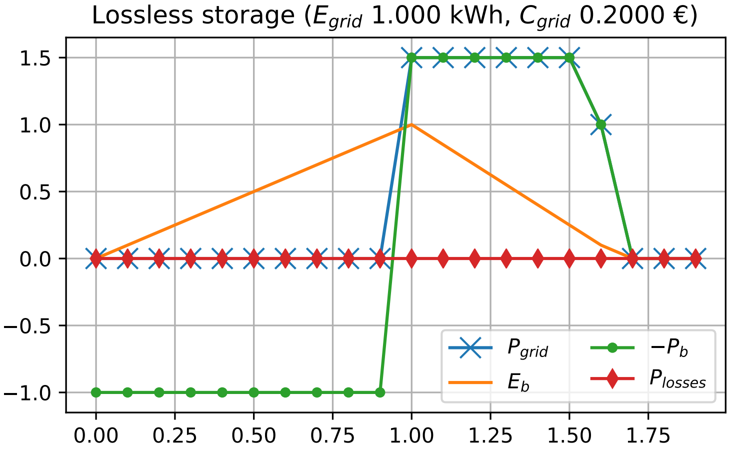
Experiment objective

See the effect of the loss model on the charge/discharge profile

Remark: All loss models calibrated for same 80% round-trip storage efficiency on the experiment

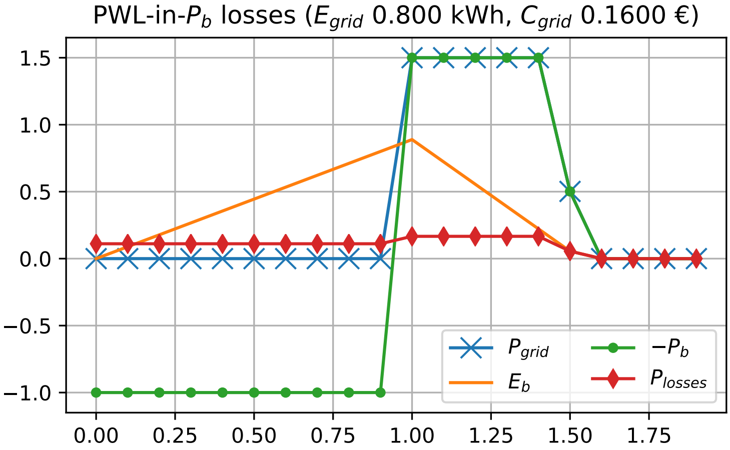
Production shifting experiment

Introductory case: lossless storage



Production shifting experiment

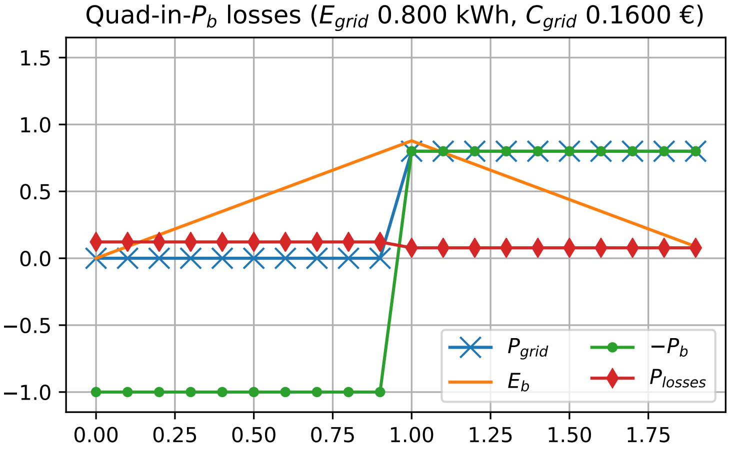
Case 1: PWL-in-P model (constant efficiency)



PWL-in-P losses reduce the profit, but no other effect on the profile

Production shifting experiment

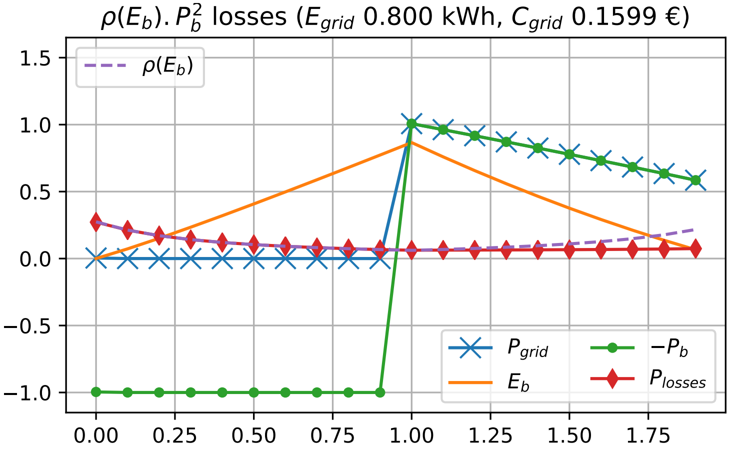
Case 2: Quadratic-in-P model



Quadratic losses *smooth out* the charge/discharge power

Production shifting experiment

Case 3: P^2/E model (supercapacitor) → losses depend on State of Energy



Discharge power is reduced at low State of Energy

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Conclusion

Contribution:

- Unified description of storage loss relaxation
  (linear & nonlinear cases scattered in previous literature)
- One "convex monomial loss model" model to bind them all

Impact

More elaborate loss models unleash more realistic storage trajectories

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Future work:

- Characterize the worst-case amount of artificially wasted energy, when the relaxation fails (negative energy price)
- Fit the "convex monomial loss model" to actual Lithium-ion battery data