# Stochastic Optimal Control for an Energy Storage example: smoothing Ocean Wave Energy

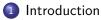
Pierre Haessig

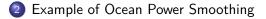
Supélec — IETR pierre.haessig@supelec.fr

Lab seminar with Bart De Schutter, November 20, 2014

Based on: P. Haessig, T. Kovaltchouk, B. Multon, H. Ben Ahmed, S. Lascaud "Computing an Optimal Control Policy for an Energy Storage", EuroSciPy 2013 http://arxiv.org/abs/1404.6389

## Outline of the presentation







Using Dynamic Programming

# Outline of the presentation



#### Introduction

2 Example of Ocean Power Smoothing

3) Using Dynamic Programming

## My background

Pierre Haessig

- Assistant professor at Supélec since September 2014
- PhD on Electricity Storage in relation to Wind Energy (control & sizing), with SATIE lab (ENS Rennes) and EDF R&D.



# StoDynProg: a software package for Dynamic Opt.

Working on the management of Energy Storage with Wind Power, I've progressively discovered that:

- my problems fall in the class of *Dynamic Optimization* (a quite specific problem structure)
- the Dynamic Programming approach exists to solve them.
- basic DP algorithms are "too simple to be worth implementing" !!

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So I've started a generic code to solve all *my* problems and hopefully other Dynamic Optimization problems as well.

I wanted to challenge this "genericity claim" by trying it on a *different* problem: I took it from a topic of interest of my research group: Ocean Power Smoothing (with an Energy Storage).

## Outline of the presentation





#### (2) Example of Ocean Power Smoothing

# Ocean Wave Energy Harvesting



(CC-BY-NC picture by polandeze) www.flickr.com/photos/polandeze/3151015577

Harvesting electric power from Ocean Waves with "big machines" is an active area of Research & Development.

There are no industrialized devices yet (unlike for wind & sun), but rather a wide variety of prototypes machines: Wave Energy Converters



E.ON P2 Pelamis, July 2011 http://www.pelamiswave.com

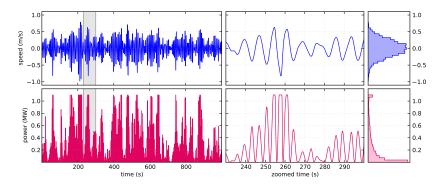
# Ocean Energy Converter: the SEAREV



Hydro-mechanical design from Centrale Nantes.

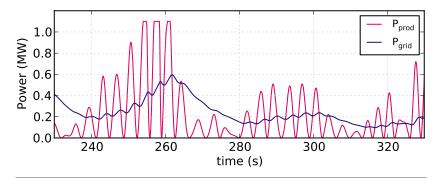
Multon & Ben Ahmed group (SATIE) involved in the electric generator design.

# Ocean Energy Converter: the SEAREV a highly fluctuating output



SEAREV is a giant double-pendulum that swings with the waves. An electric generator "brakes" the inner wheel to generates power  $(P_{prod} = T(\Omega) \times \Omega)$ .

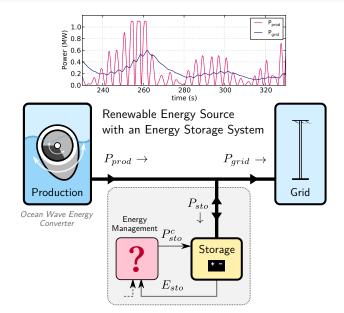
#### Power smoothing

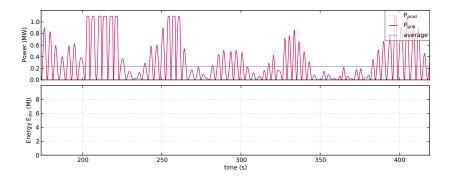


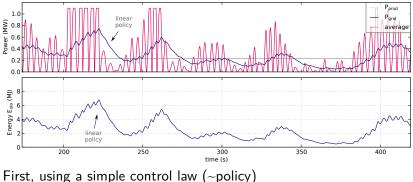
#### Objective of this application

I want to smooth out the variations of the power production. This requires an **energy buffer** to store the difference  $(P_{prod} - P_{grid}).$ 

## Power smoothing using an Energy Storage

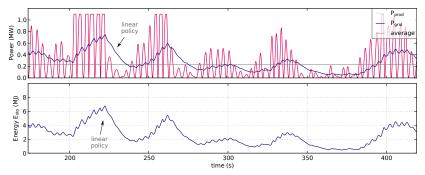






$$P_{grid}(t) = rac{P_{max}}{E_{rated}} E_{sto}(t)$$

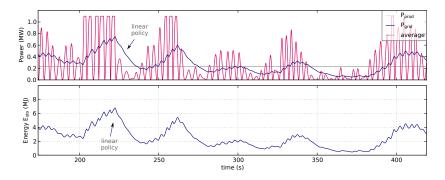
... quite good result but storage is underused  $\rightarrow$  could do better.



"Doing better" is defined with an additive cost function which penalizes  $P_{grid}$  variations:

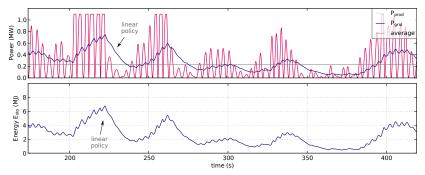
$$J = rac{1}{N} \mathbb{E} \left\{ \sum_{k=0}^{N-1} cost(P_{grid}(k) - P_{avg}) 
ight\} \quad ext{with } N o \infty$$

cost J should be minized.



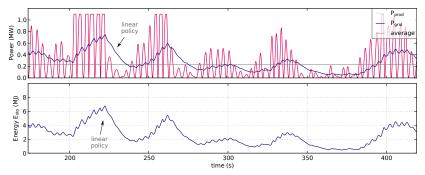
Controlling the storage (choosing  $P_{grid}$  at each time step) in order to minimize a cost function is a **Stochastic Dynamic Optimization** problem

(also called Stochastic Optimal Control)



*Dynamic Programming* (Richard Bellman, ~1950) teaches us that the optimal control is a *state feedback* policy:

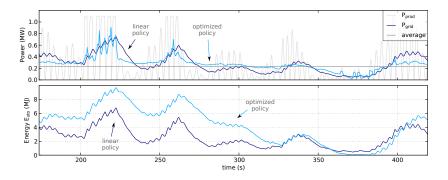
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 with  $x = (E_{sto}, other variables?)$ 



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And DP gives us *methods to compute* this policy function  $\mu$ ...



And now applying the optimal feedback policy  $\mu^*$ , the standard deviation of the power injected to the grid is reduced by ~20 % compared to the heuristic policy.

This improvement just comes from a smarter use of the stored energy.

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Using Dynamic Programming

#### Model for the dynamics

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$$\Omega(k) = \phi_1 \Omega(k-1) + \phi_2 \Omega(k-2) + w(k)$$

- $\mathsf{AR}(2) \to \mathsf{state}$  space, with speed  $\Omega$  and acceleration A
- Non-linear transform gives the power:  $P_{prod} = \mathcal{T}(\Omega) imes \Omega$
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Full state: 
$$x = (E_{sto}, \Omega, A)$$

# Dynamic Programming equation

In the end, the optimization problem turns into solving the DP equation:

$$J^* + \tilde{J}(x) = \min_{u \in U(x)} \mathbb{E} \left\{ \underbrace{cost(x, u, w)}_{\text{instant cost}} + \underbrace{\tilde{J}(f(x, u, w))}_{\text{cost of the future}} \right\}$$

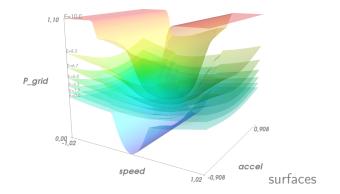
u is control and w is random perturbation, using generic notations

- It is a *functional* equation: should be solved for all x
- The optimal policy  $\mu: x \mapsto u$  appears as the argmin.

The DP equation is solved on a **discrete grid** over the state space. With  $x \in \mathbb{R}^n$ ,  $\tilde{J}$  and  $\mu$  are computed as *n*-dim. arrays.

# The optimal policy $P_{grid}(E_{sto}, speed, accel)$

Optimal control is a  $\mathbb{R}^3 \mapsto \mathbb{R}$  function (i.e. numerically, a 3D array)



 $P_{grid}(speed, accel)$ , for different levels of energy  $E_{sto}$ 

#### Conclusion

About Dynamic Programming (DP) interest

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#### Code and data openly available on GitHub

https://github.com/pierre-haessig/stodynprog/tree/master/examples/