

# Two time scales stochastic dynamic optimization

Managing energy storage investment, aging and operation in microgrids

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# Optimization for microgrids with storage

Microgrids control architecture is often constituted of multiple levels handling multiple time scales

Energy storage management requires to deal with uncertainty and information dynamic

We use two time scales stochastic dynamic optimization to model two control levels and their interaction



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Efficacy

efficacy

# Outline

- 1 Introduction: Electrical storage management in microgrids
  - Storage control in a microgrid
  - Hierarchical control architecture of microgrids
- 2 Modeling: Managing intraday arbitrage, aging and renewal
  - Two time scales management: investment/arbitrage
  - Intraday arbitrage problem statement
  - Long term aging/investment problem statement
  - Two time scales stochastic optimization problem
- 3 Solving: Decomposition method and numerical results
  - Decomposition method
  - Numerical results



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# Storage control in a microgrid

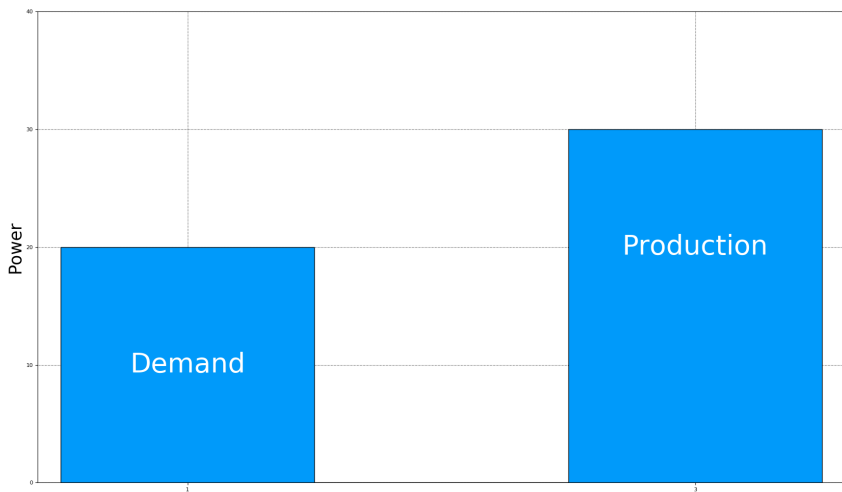


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# Why storage in a microgrid?

Ensure supply demand balance without wastes or curtailment:



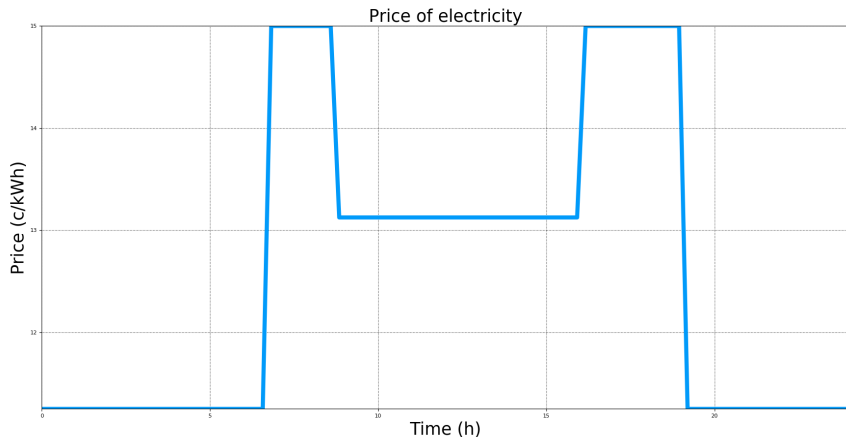
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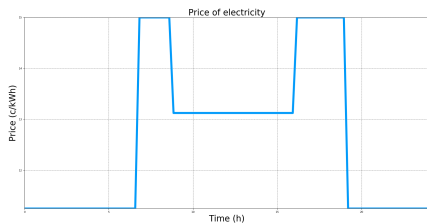
# Why storage in a microgrid?

Energy tariff arbitrage and ancillary services

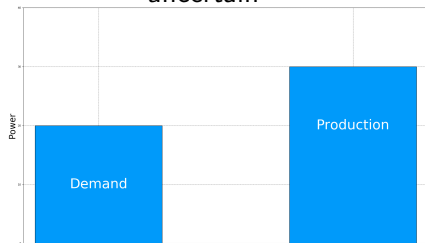


# Why stochastic dynamic optimization?

Price of electricity might be uncertain

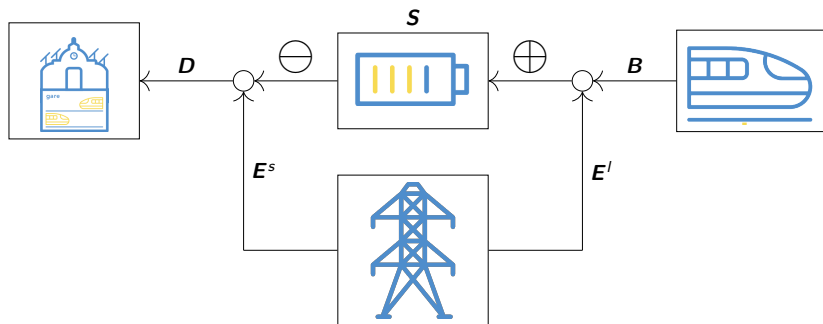


Demand and production are uncertain





# Subway station microgrid example



# Hierarchical control architecture of microgrids

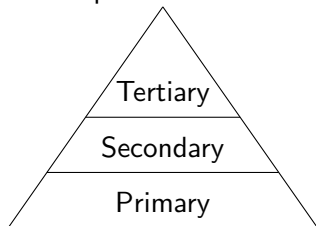


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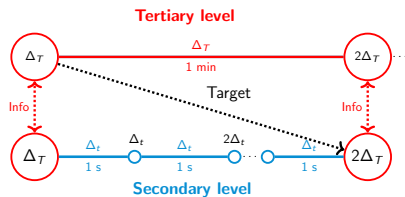


# A way to deal with multiple time scales

Multiple control levels



To handle multiple time scales



## Small time scale: voltage stability of the grid

- Objective: voltage stability of the grid
- Time step: 1s
- Horizon: 1 min
- Input from superior level: storage input/output energy target every minute
- Output: effective command for storage every second

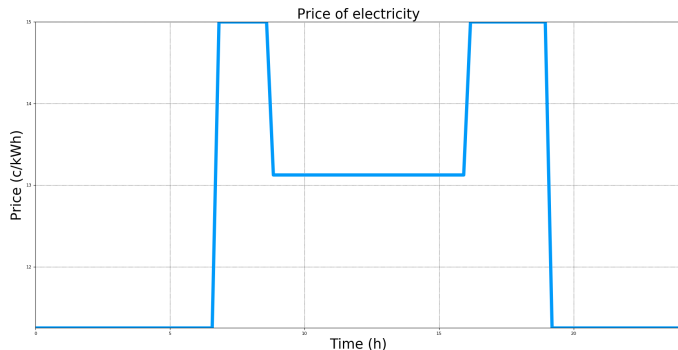


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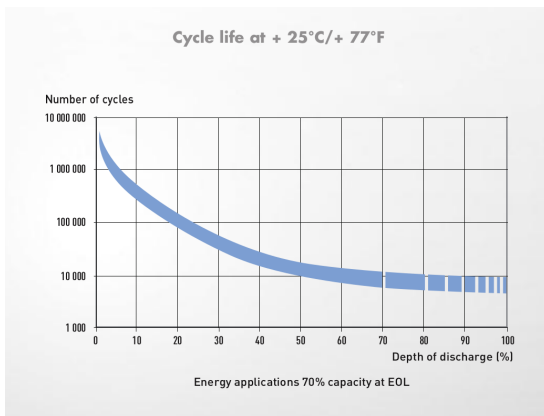
## Medium time scale: intraday energy tariff arbitrage

- Objective: energy intraday arbitrage
- Time step: 1 min
- Horizon: 24h
- Input from superior level: storage aging target everyday
- Output to inferior level: storage input/output energy target every minute



# Large time scale: long term aging and investments strategy

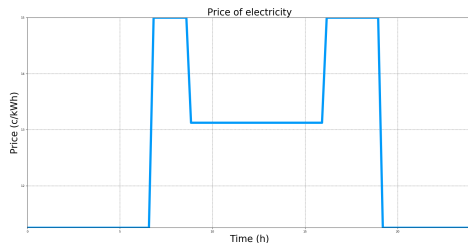
- Objective: storage long term economic profitability
- Time step: 1 day
- Horizon: 10 years
- Output to inferior level: storage aging target every day



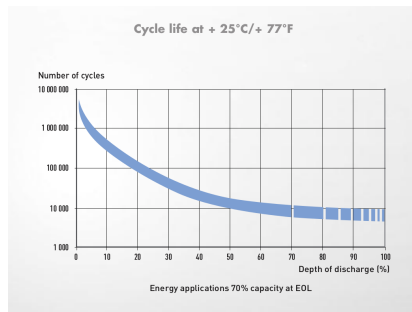
# Structure of the talk

We focus on medium and large levels interaction to optimize storage:

## Intraday energy arbitrage



## Long term aging



SAFT intensium max technical sheet



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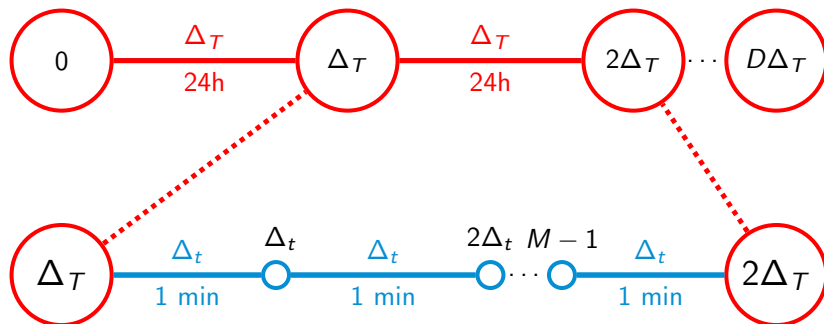


# Two time scales management: investment/arbitrage



# Two time scales

## Long term aging and renewal

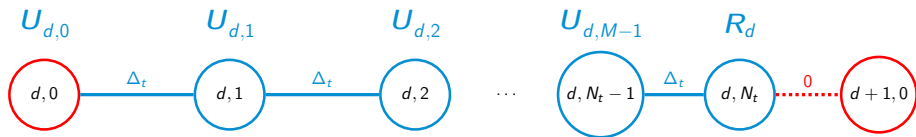


## Intraday arbitrage



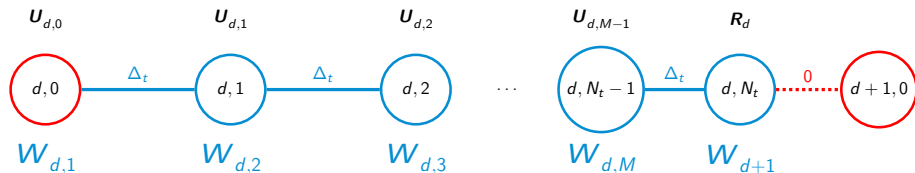
# We make decisions every minutes $m$ and every day $d$

- **Day  $d$ , Minute  $m$ :** How much energy  $U_{d,m}$  do I charge or discharge from my current battery with capacity  $C_d$ ?
- At the end of **Day  $d$**  should I buy a new battery with capacity  $R_d$ ?



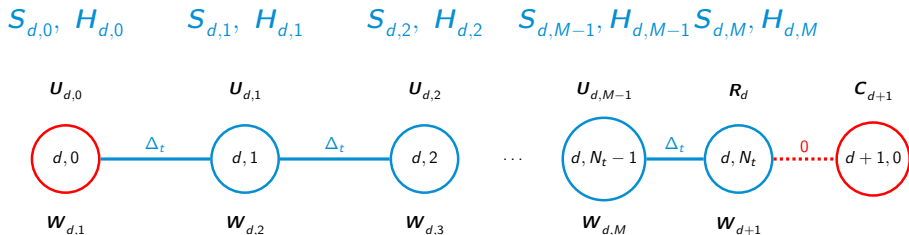
# Uncertain events occur right after we made our decisions

- **Day  $d$** , end of **Minute  $m$** : we observe how much intermittent energy  $W_{d,m+1}$  we receive
- At the end of **Day  $d$**  we observe the batteries cost  $W_{d+1}$  on the market



# Decisions and uncertainty impact state variables

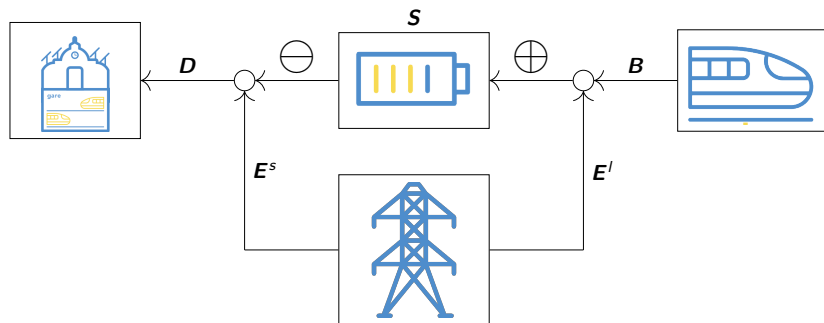
- **Day  $d$** , end **Minute  $m$** : decision  $\mathbf{U}_{d,m}$  and realization  $\mathbf{W}_{d,m+1}$  change our battery state of charge  $\mathbf{S}_{d,m}$  to  $\mathbf{S}_{d,m+1}$  and our battery state of health  $\mathbf{H}_{d,m}$  to  $\mathbf{H}_{d,m+1}$
- At the end of **Day  $d$**  decision  $\mathbf{R}_d$  change our battery capacity  $\mathbf{C}_d$  to  $\mathbf{C}_{d+1}$



# Intraday arbitrage problem statement



# Representation of the subway station problem



## Station node

- $D$ : Demand station
- $E^s$ : From grid to station
- $\ominus$ : Discharge battery

## Subways node

- $B$ : Braking
- $E'$ : From grid to battery
- $\oplus$ : Charge battery



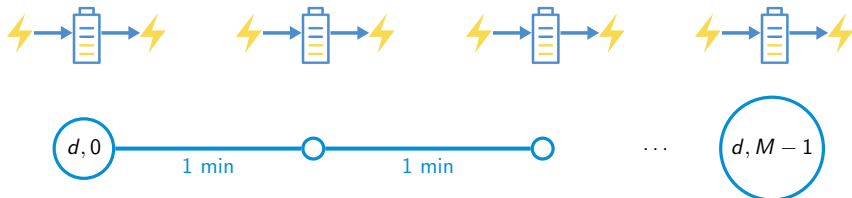
# Battery state of charge dynamics

For a given charge/discharge strategy  $\mathbf{U}$  over a day  $d$ :

$$\mathbf{S}_{d,m+1} = \mathbf{S}_{d,m} - \underbrace{\frac{1}{\rho_d} \mathbf{U}_{d,m}^-}_{\ominus} + \underbrace{\rho_c \text{sat}(\mathbf{S}_{d,m}, \mathbf{U}_{d,m}^+, \mathbf{B}_{d,m+1})}_{\oplus}$$

with

$$\text{sat}(x, u, b) = \min\left(\frac{S_{\max} - x}{\rho_c}, \max(u, b)\right)$$





# Battery aging dynamics

For a given charge/discharge strategy  $\mathbf{U}$  over a day  $d$

$$\mathbf{H}_{d,m+1} = \mathbf{H}_{d,m} - \frac{1}{\rho_d} \mathbf{U}_{d,m}^- - \rho_c \text{sat}(\mathbf{S}_{d,m}, \mathbf{U}_{d,m}^+, \mathbf{B}_{d,m+1})$$



# Every minute we save energy and money

If we have a battery on day  $d$  and minute  $m$  we save:

$$p_{d,m}^e \left( \underbrace{E_{d,m+1}^s + E_{d,m+1}^l - D_{d,m+1}}_{\text{Saved energy}} \right)$$

$p_{d,m}^e$  is the cost of electricity on day  $d$  at minute  $m$



# Summary of short term/Fast variables model

We call, at day  $d$  and minute  $m$ ,

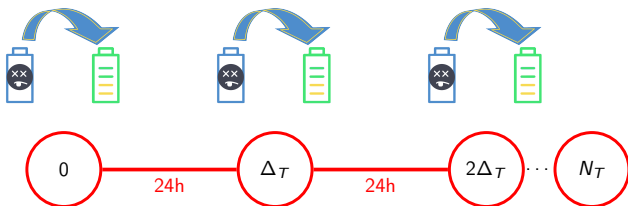
- fast state variables:  $\mathbf{X}_{d,m}^f = \begin{pmatrix} \mathbf{S}_{d,m} \\ \mathbf{H}_{d,m} \end{pmatrix}$
- fast decision variables:  $\mathbf{U}_{d,m}^f = \begin{pmatrix} \mathbf{U}_{d,m}^- \\ \mathbf{U}_{d,m}^+ \end{pmatrix}$
- fast random variables:  $\mathbf{W}_{d,m}^f = \begin{pmatrix} \mathbf{B}_{d,m} \\ \mathbf{D}_{d,m} \end{pmatrix}$
- fast cost function:  $L_{d,m}^f(\mathbf{X}_{d,m}^f, \mathbf{U}_{d,m}^f, \mathbf{W}_{d,m+1}^f)$
- fast dynamics:  $\mathbf{X}_{d,m+1}^f = F_{d,m}^f(\mathbf{X}_{d,m}^f, \mathbf{U}_{d,m}^f, \mathbf{W}_{d,m+1}^f)$



# Long term aging/investment problem statement



# We decide our battery purchases at the end of each day



Should we replace our battery  $\mathbf{C}_d$  by buying a new one  $\mathbf{R}_d$  or not?

$$\mathbf{C}_{d+1} = \begin{cases} \mathbf{R}_d, & \text{if } \mathbf{R}_d > 0 \\ f(\mathbf{C}_d, \mathbf{H}_{d,M}), & \text{otherwise} \end{cases}$$

paying renewal cost  $\mathbf{P}_d^b \mathbf{R}_d$  at uncertain market prices  $\mathbf{P}_d^b$

# Summary of long term/Slow variables model

We call, at day  $d$ ,

- slow state variables:  $\mathbf{X}_d^s = (\mathbf{C}_d)$
- slow decision variables:  $\mathbf{U}_d^s = (\mathbf{R}_d)$
- slow random variables:  $\mathbf{W}_d^s = (\mathbf{P}_d^b)$
- slow cost function:  $L_d^s(\mathbf{X}_d^s, \mathbf{U}_d^s, \mathbf{W}_{d+1}^s) = \mathbf{P}_d^b \mathbf{R}_d$
- slow dynamics:  $\mathbf{X}_{d+1}^s = F_d^s(\mathbf{X}_d^s, \mathbf{U}_d^s, \mathbf{W}_{d+1}^s)$



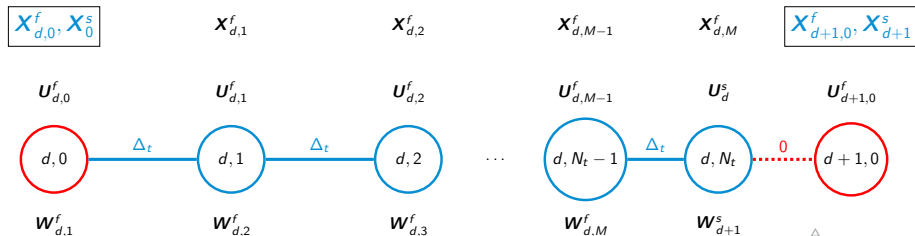
## A link between days

The initial "fast state" at the beginning of day  $d$  deduces from:

$$\mathbf{x}_{d,0}^f = \phi_d(\mathbf{x}_d^s, \mathbf{x}_{d-1,M}^f)$$

The initial "slow state" at the beginning of day  $d+1$  deduces from all that happened the previous day:

$$\mathbf{x}_{d+1,0}^s = F_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f)$$



# We formulate a two time scales stochastic optimization problem



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We minimize fast and slow costs over the long term

$$\min_{\mathbf{x}^f, \mathbf{x}^s, \mathbf{u}^f, \mathbf{u}^s} \mathbb{E} \left[ \sum_{d=0}^{M-1} \left( \sum_{m=0}^{M-1} L_{d,m}^f(\mathbf{x}_{d,m}^f, \mathbf{u}_{d,m}^f, \mathbf{w}_{d,m+1}^f) \right) + L_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f) \right]$$

$$\mathbf{x}_{d,m+1}^f = F_{d,m}^f(\mathbf{x}_{d,m}^f, \mathbf{u}_{d,m}^f, \mathbf{w}_{d,m+1}^f)$$

$$\mathbf{x}_{d,0}^f = \phi_d(\mathbf{x}_d^s, \mathbf{x}_{d-1,M}^f)$$

$$\mathbf{x}_{d+1}^s = F_d^s(\mathbf{x}_d^s, \mathbf{u}_d^s, \mathbf{w}_{d+1}^s, \mathbf{x}_{d,0}^f, \mathbf{u}_{d,:}^f, \mathbf{w}_{d,:}^f)$$

$$\mathbf{u}_{d,m}^f \preceq \mathcal{F}_{d,m}$$

$$\mathbf{u}_d^s \preceq \mathcal{F}_{d,M}$$



# Stochastic optimal control reformulation

We call

$$\mathbf{X}_d = (\mathbf{X}_{d,0}^f, \mathbf{X}_d^s)$$

$$\mathbf{U}_d = (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s)$$

$$\mathbf{W}_d = (\mathbf{W}_{d-1,:}^f, \mathbf{W}_d^s)$$

we can reformulate the problem as

$$\min_{\mathbf{X}, \mathbf{U}} \mathbb{E} \left[ \sum_{d=0}^{M-1} L_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \right]$$

$$\mathbf{X}_{d+1} = F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1})$$

$$\mathbf{U}_{d,m}^f \preceq \mathcal{F}_{d,m}$$

$$\mathbf{U}_d^s \preceq \mathcal{F}_{d,M}$$

where the non-anticipativity constraints are not standard

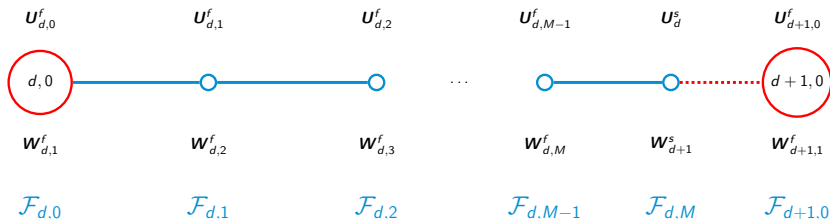


# Information flow model

$$\mathcal{F}_{d,m} = \sigma \begin{pmatrix} \mathbf{W}_{d',m'}^f, & d' < d, & m' \leq M+1 \\ & \mathbf{W}_{d'}^s, & d' \leq d \\ \mathbf{W}_{d,m'}^f, & & m' \leq m \end{pmatrix} = \sigma \begin{pmatrix} \text{previous days fast noises} \\ \text{previous days slow noises} \\ \text{current day previous minutes fast noises} \end{pmatrix}$$

$$\mathbf{X}_d = (\mathbf{X}_{d,0}^f, \mathbf{X}_0^s)$$

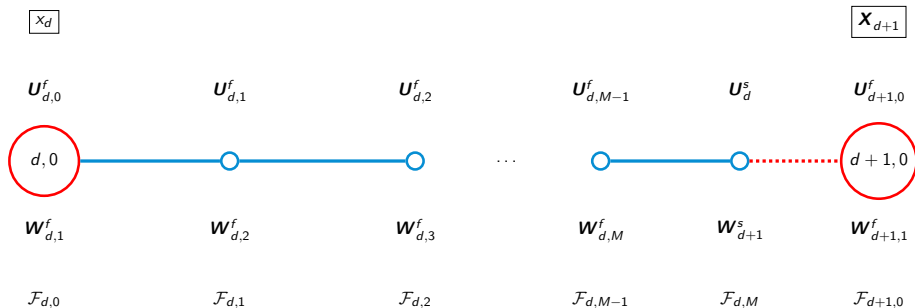
$$\mathbf{X}_{d+1} = (\mathbf{X}_{d+1,0}^f, \mathbf{X}_{d+1}^s)$$



# We can write a dynamic programming equation

When the  $W_d$  are independent

$$V_d(x_d) = \min_{U_d} \mathbb{E} [L_d(x_d, U_d, W_{d+1}) + V_{d+1}(X_{d+1})]$$



# With value functions defined inductively

Every day  $d$ , we can define a **value function** that factorizes as **function of the state  $\mathbf{X}_d$**  if the  $\mathbf{W}_d$  are independent.

$$V_d(x_d) = \min_{\mathbf{x}_{d+1}, \mathbf{U}_d} \mathbb{E} \left[ L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) + V_{d+1}(\mathbf{X}_{d+1}) \right]$$

$$\text{s.t } \mathbf{X}_{d+1} = F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1})$$

$$\mathbf{U}_{d,m}^f \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:m}^f)$$

$$\mathbf{U}_d^s \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:M}^f)$$

$$\mathbf{U}_d = (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s)$$

$$\mathbf{X}_d = x_d$$

The value of the whole problem being:  $V_0(x_0)$ .



How to decompose the problem  
into  
a **daily optimization problem**  
and  
an **intraday optimization problem?**



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# Let's "split" the min

$$V_d(x_d) = \min_{\mathbf{X}_{d+1}} \min_{\mathbf{U}_d} \mathbb{E} \left[ L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) + V_{d+1}(\mathbf{X}_{d+1}) \right]$$

s.t

$$\mathbf{X}_{d+1} = F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1})$$
$$\mathbf{U}_{d,m}^f \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:m}^f)$$
$$\mathbf{U}_d^s \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:M}^f)$$
$$\mathbf{U}_d = (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s)$$
$$\mathbf{X}_d = x_d$$





## Let's introduce an auxiliary variable

$$V_d(x_d) = \min_{\mathbf{Y}_{d+1}} \min_{\mathbf{X}_{d+1}} \min_{\mathbf{U}_d} \mathbb{E} \left[ L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) + V_{d+1}(\mathbf{X}_{d+1}) \right]$$

$$\text{s.t. } \mathbf{X}_{d+1} = \mathbf{Y}_{d+1}$$

$$F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) = \mathbf{Y}_{d+1}$$

$$\mathbf{U}_{d,m}^f \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:m}^f)$$

$$\mathbf{U}_d^s \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:M}^f)$$

$$\mathbf{U}_d = (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s)$$

$$\mathbf{X}_d = x_d$$

$$\mathbf{Y}_{d+1} \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d+1})$$



# Let's "distribute" the mins

$$V_d(x_d) = \min_{\mathbf{Y}_{d+1}} \left[ \min_{\mathbf{U}_d} \mathbb{E} L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) + \min_{\mathbf{X}_{d+1}} \mathbb{E} V_{d+1}(\mathbf{X}_{d+1}) \right]$$

$$\text{s.t. } \mathbf{X}_{d+1} = \mathbf{Y}_{d+1}$$

$$F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) = \mathbf{Y}_{d+1}$$

$$\mathbf{U}_{d,m}^f \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:m}^f)$$

$$\mathbf{U}_d^s \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:M}^f)$$

$$\mathbf{U}_d = (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s)$$

$$\mathbf{X}_d = x_d$$

$$\mathbf{Y}_{d+1} \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d+1})$$



## The intraday arbitrage problem appears

For a given  $\mathbf{Y}_{d+1} \in L^0(\Omega, \mathcal{F}, \mathbb{P})$ , with  $\sigma(\mathbf{Y}_{d+1}) \subset \sigma(\mathbf{X}_d, \mathbf{W}_{d+1})$ ,

$$\begin{aligned}\phi_d(x_d, [\mathbf{Y}_{d+1}]) &= \min_{\mathbf{U}_d} \mathbb{E} L_d(x_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \\ \text{s.t } F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) &= \mathbf{Y}_{d+1} \\ \mathbf{U}_{d,m}^f &\preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:m}^f) \\ \mathbf{U}_d^s &\preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d,1:M}^f) \\ \mathbf{U}_d &= (\mathbf{U}_{d,:}^f, \mathbf{U}_d^s) \\ \mathbf{X}_d &= x_d\end{aligned}$$

We use the notation  $f([\mathbf{W}])$  to emphasize that  $f$ 's domain is  $L^0(\Omega, \mathcal{F}, \mathbb{P})$ .

This is the intraday arbitrage problem with stochastic final age target!



## Back to the expression of the daily value functions

As  $\mathbf{Y}_{d+1} = \mathbf{X}_{d+1}$  we obtain:

$$V_d(x_d) = \min_{\mathbf{X}_{d+1}} \left[ \overbrace{\phi_d(x_d, [\mathbf{X}_{d+1}])}^{\text{intraday problem}} + \overbrace{\mathbb{E} V_{d+1}(\mathbf{X}_{d+1})}^{\text{expected cost to go}} \right]$$
$$\text{s.t } \mathbf{X}_{d+1} \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d+1})$$

with  $\phi_d(x_d, [\mathbf{X}_{d+1}]) = +\infty$  if  $\mathbf{X}_{d+1}$  is an unreachable target for the intraday problem.



# Significant difficulties remain

- Computing  $\phi_d(x_d, [\mathbf{X}_{d+1}])$  for every  $\mathbf{X}_{d+1}$  is very expensive
- Solving the intraday problem with a stochastic final target is hard ( $\mathbf{X}_{d+1} \preceq \sigma(\mathbf{X}_d, \mathbf{W}_{d+1})$ )

Then why is it interesting?

- We can solve the intraday problem  $\phi_d$  with another method (DP, SDDP, SP, PH, MPC)
- We can exploit the problem periodicity ( $\forall d, \phi_d = \phi_0$ )
- We can simplify measurability ( $\mathbf{X}_{d+1} \preceq \sigma(\mathbf{X}_d)$ )
- We can exploit value functions monotonicity (relax the coupling constraint  $F_d(\mathbf{X}_d, \mathbf{U}_d, \mathbf{W}_{d+1}) \geq \mathbf{X}_{d+1}$ ) [2]

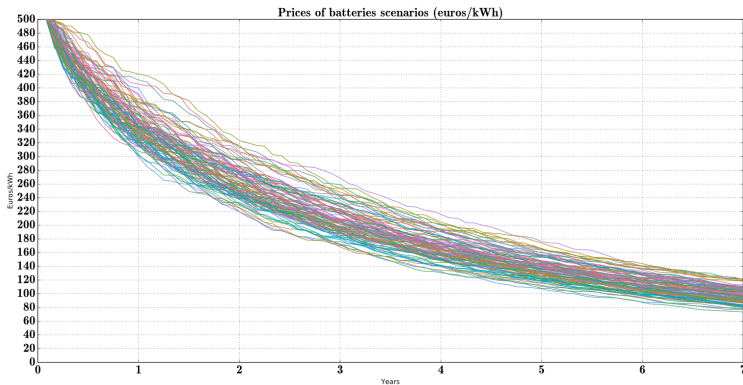


# Numerical results



# Synthetic price of batteries data

- Batteries cost stochastic model: synthetic scenarios that approximately coincide with market forecasts



# Net Present Value

- 7 years horizon
- Yearly discount factor = 0.95
- 10,000  $C^b$  scenarios to model randomness
- 1 buying/aging decision per month
- 1 charge/discharge decision every 15 min
- Constraint: having a battery everytime with at least one cycle a day

**Objective:** maximize expected discounted revenues over 7 years





# Numerical method: Intraday DP + Extraday DP

We use DP for intraday decisions and DP for daily decisions.

Simplifications:

- Monotonicity
- Daily periodicity
- $\mathbf{X}_{d+1} \preceq \sigma(\mathbf{X}_d)$ : We decide aging at the beginning of the day



# Results

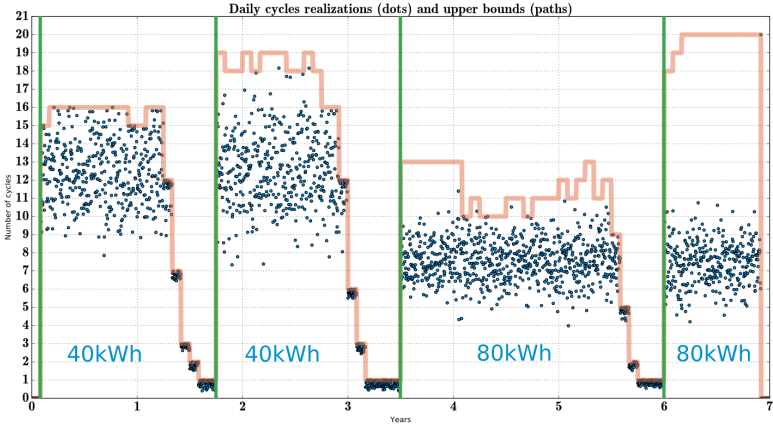
	<b>SDP</b>	<b>SDP + SDP</b>
Offline comp. time	$\infty$	1 min + 15 min
Simulation comp. time	?	[25s,30s]
Upper bound	?	+128k

In Julia with a Core I7, 1.7 Ghz, 8Go ram + 12Go swap SSD



# 1 simulation: cycles

NPV = 80,000 euros



# Conclusion and ongoing work

Our study leads to the following conclusions:

- Controlling aging is relevant
- The method can be used for aging aware intraday control as well as investment management
- This modeling framework allows to find methods to solve multi time scales problems

We are now focusing on

- Improving risk modelling
- Improving batteries cost stochastic model
- Improving aging model with capacity degradation
- Applying dual decomposition methods



# References



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